

# A GREEDY BLIND CALIBRATION METHOD FOR COMPRESSED SENSING WITH UNKNOWN SENSOR GAINS

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## ABSTRACT

The realisation of sensing modalities based on the principles of compressed sensing is often hindered by discrepancies between the mathematical model of its sensing operator, which is necessary during signal recovery, and its actual physical implementation, whose values may differ significantly from the assumed model. In this paper we tackle the bilinear inverse problem of recovering a sparse input signal and some unknown, unstructured multiplicative factors affecting the sensors that capture each compressive measurement. Our methodology relies on collecting a few snapshots under new draws of the sensing operator, and applying a greedy algorithm based on projected gradient descent and the principles of iterative hard thresholding. We explore empirically the sample complexity requirements of this algorithm by testing the phase transition of our algorithm, and show in a practically relevant instance of compressive imaging that the exact solution can be obtained with only a few snapshots.

**Index Terms**— Compressed Sensing, Blind Calibration, Iterative Hard Thresholding, Non-Convex Optimisation, Bilinear Inverse Problems

## 1. INTRODUCTION

The implementation of practical sensing schemes based on Compressed Sensing (CS) [1] often encounters physical non-idealities in realising the mathematical model of the sensing operator, whose accuracy is crucial to attaining a high-quality recovery of the observed signal [2, 3]. Among such non-idealities, we here focus on the case in which each compressive measurement is affected by an unknown multiplicative factor or *sensor gain*, i.e., on the sensing model

$$\mathbf{y}_l = \text{diag}(\mathbf{g})\mathbf{A}_l\mathbf{x}, \quad l \in [p] := \{1, \dots, p\}, \quad (1)$$

where  $\mathbf{x} \in \mathbb{R}^n$  is the input signal,  $\mathbf{A}_l \in \mathbb{R}^{m \times n}$ ,  $l \in [p]$  are independent and identically distributed (i.i.d.) random sensing

matrices, and  $\mathbf{y}_l \in \mathbb{R}^m$ ,  $l \in [p]$  are the respective *snapshots* of measurements obtained by applying each sensing matrix to  $\mathbf{x}$  (the reason why the acquisition is partitioned in snapshots will be cleared below). In this *uncalibrated* sensing model  $\mathbf{g} \in \mathbb{R}_+^m$  is an unknown set of positive-valued gains that remains identical throughout the snapshots, but whose value is unknown. Hence, this sensing model is *bilinear* in  $\mathbf{x}$  and  $\mathbf{g}$ , and retrieving both quantities given the measurements is a non-trivial inverse problem.

In previous contributions [4, 5] we have shown that instances of (1) with sensing matrices having i.i.d. sub-Gaussian entries (for a rigorous definition, see [6]) and unstructured  $(\mathbf{x}, \mathbf{g})$  can be provably solved by a simple, suitably initialised projected gradient descent on a non-convex objective. The number of measurements that ensures the recovery of the exact solution was shown to be<sup>1</sup>  $mp \gtrsim n + m$ , i.e., a linear *sample complexity* in the number of unknowns (up to log factors; this is attained in [5]).

In this paper we extend our study to the case in which the single input signal  $\mathbf{x}$  is sparse; to leverage this model on  $\mathbf{x}$  we simply resort to a hard thresholding operator at each iterate of our former non-convex algorithm, turning it into a greedy scheme. The proposed greedy approach allows for blind calibration in actual CS schemes; the additional requirement of our methodology is a set of  $p$  snapshots that collects a sufficient amount of information on  $(\mathbf{x}, \mathbf{g})$ . Our emphasis is on assessing, at least numerically, whether the sample complexity can be reduced in function of  $k$  (up to log factors). Hence, provided  $\mathbf{x}$  is sufficiently sparse, we will show empirically that the total amount of measurements  $mp$  can be lower than  $n$  while still recovering both  $(\mathbf{x}, \mathbf{g})$ .

Our framework could be practically applied to compressive imaging schemes [7–10], specially those that feature snapshot acquisition by convolving an input signal with one or more random masks. When the sensor gains are not calibrated, e.g., in the presence of fixed-pattern noise or strong pixel-response non-uniformity [11], taking a few snapshots will allow for on-line blind calibration without missing any

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<sup>1</sup>Hereafter, given two functions  $f, g$ ,  $f \gtrsim g$  indicates that  $f > Cg$  for some constant  $C > 0$ .

instance of the signal  $\mathbf{x}$  in the process.

### 1.1. Related Work

Blind calibration of sensor gains has been tackled in the literature prior to CS; an initial formulation of this gain calibration problem for sensor networks was introduced in [12, 13]. In the context of CS, many algorithms have been proposed to cope with such model errors [14–19]. Interestingly, most algorithms focus on the use of sparse or known subspace models for several input signals, rather than random draws of the sensing operator itself (which is typically possible in CS schemes). However efficient, most of the referenced methods have no theoretical guarantee of recovering the exact solution.

On the other hand, an algorithm using a single sparse input signal with provable guarantees was first introduced by Ling and Strohmer [20] based on a *lifting* approach to the problem (similarly to [21]). The main drawback of such an approach is computational, given that it corresponds to solving a very large-scale semidefinite programming problem.

Our former contribution [4, 5] showed that a non-convex approach could provide exact recovery with theoretical guarantees and some computational advantage; it was partly inspired by the methodologies Candès *et al.* [22] and White *et al.* [23] pursued for the problem of phase retrieval.

For what concerns the related task of blind deconvolution, very recent non-convex approaches to this inverse problem adopt algorithms similar to our own [24, 25], yet targeting a more general context than blind calibration and therefore subject to different requirements and conditions than those we encountered independently in our method.

### 1.2. Contributions and Outline

Our paper extends the non-convex algorithm devised in [4, 5] to account for a sparse model in the signal domain; this is a fundamental prior for CS, whereas sparse models on the gains  $\mathbf{g}$  could be inapplicable when these are drawn at random as each sensor is manufactured. Thus, we adopt a greedy algorithm to enforce signal-domain sparsity, and detail by numerical experiments its performances as a function of the problem dimensions.

Our findings are presented as follows. In Section 2 we introduce the non-convex problem and propose a greedy algorithm based on hard thresholding. This algorithm is studied numerically in Section 3, where we focus on the empirical phase transition as the problem dimensions vary. We then simulate a practical case of blind calibration for compressive imaging in Section 4. A conclusion is drawn afterwards.

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### Algorithm 1 Blind Calibration with Iterative Hard Thresholding.

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- 1: Initialise  $\xi_0 := \frac{1}{mp} \sum_{l=1}^p (\mathbf{A}_l)^\top \mathbf{y}_l$ ;  $\gamma_0 := \mathbf{1}_m$ ; the exact sparsity level  $k$ ;  $j := 0$ .
  - 2: **while** stop criteria not met **do**
  - 3:   {Line searches}  
 $\mu_\xi := \operatorname{argmin}_{v \in \mathbb{R}} f(\xi_j - v \nabla_\xi f(\xi_j, \gamma_j), \gamma_j)$   
 $\mu_\gamma := \operatorname{argmin}_{v \in \mathbb{R}} f(\xi_j, \gamma_j - v \nabla_\gamma^\perp f(\xi_j, \gamma_j))$
  - 4:   {Hard-thresholded signal estimate}  
 $\xi_{j+1} := \mathcal{ZH}_k[\mathbf{Z}^\top (\xi_j - \mu_\xi \nabla_\xi f(\xi_j, \gamma_j))]$
  - 5:   {Gain estimate}  
 $\gamma_{j+1} := \mathcal{P}_{\mathcal{G}_\rho}[\gamma_j - \mu_\gamma \nabla_\gamma^\perp f(\xi_j, \gamma_j)]$
  - 6:    $j := j + 1$
  - 7: **end while**
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## 2. A GREEDY AND NON-CONVEX APPROACH TO BLIND CALIBRATION

Our initial approach to the blind calibration problem involved defining a simple Euclidean loss,

$$f(\xi, \gamma) := \frac{1}{2mp} \sum_{l=1}^p \|\operatorname{diag}(\gamma) \mathbf{A}_l \xi - \mathbf{y}_l\|_2^2, \quad (2)$$

and solving

$$(\hat{\mathbf{x}}, \hat{\mathbf{g}}) = \operatorname{argmin}_{\xi \in \mathbb{R}^n, \gamma \in \Pi_+^m} f(\xi, \gamma) \quad (3)$$

where  $\Pi_+^m := \{\mathbf{v} \in \mathbb{R}_+^m : \mathbf{1}_m^\top \mathbf{v} = m\}$  is the *scaled probability simplex* and  $\mathbf{1}_m$  the vector of ones in  $\mathbb{R}^m$ . To begin with, up to a scaling all points in

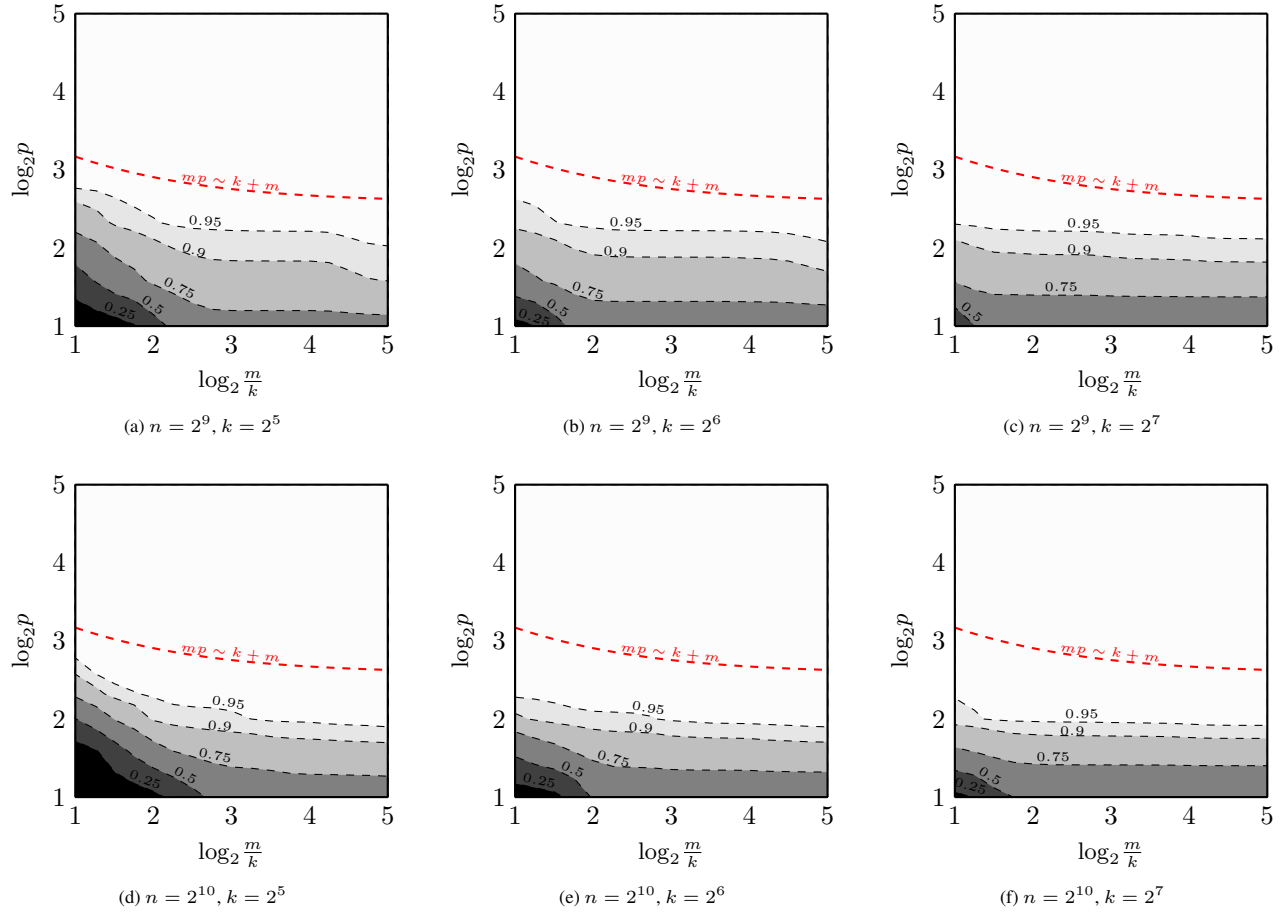
$$\{(\xi, \gamma) \in \mathbb{R}^n \times \mathbb{R}^m : \xi = \alpha \mathbf{x}, \gamma = \frac{\mathbf{g}}{\alpha}, \alpha \neq 0\}$$

are zeros of  $f(\xi, \gamma)$  (*i.e.*, the scaling of  $(\mathbf{x}, \mathbf{g})$  is anyway unrecoverable), so we adopted the constraint  $\gamma \in \Pi_+^m$  which fixes one admitted solution for  $\alpha = \frac{\|\mathbf{g}\|_1}{m}$ . This also serves to control  $\|\gamma\|_1$  during the iterates of our algorithm. We then assume that  $\mathbf{x}$  is  $k$ -sparse on an orthonormal basis  $\mathbf{Z} \in \mathbb{R}^{n \times n}$  such that  $\mathbf{x} = \mathbf{Z}\mathbf{z} \in \mathbf{Z}\Sigma_k^n$ , with  $\Sigma_k^n := \{\mathbf{u} \in \mathbb{R}^n : k = |\operatorname{supp} \mathbf{u}|\}$ . Thus, to enforce sparsity we aim to solve

$$(\hat{\mathbf{x}}, \hat{\mathbf{g}}) = \operatorname{argmin}_{\xi \in \mathbf{Z}\Sigma_k^n, \gamma \in \Pi_+^m} f(\xi, \gamma), \quad (4)$$

where the problem would be non-convex both due to the bilinear nature of  $f(\xi, \gamma)$  and to that of the union of  $k$ -dimensional canonical subspaces  $\Sigma_k^n$ . We now proceed to devise an algorithm solving (4) that accounts for the two constraints.

Firstly, if we consider that  $\|\mathbf{g}\|_1 = m$  (always verified up to a scaling) there exists a value  $\rho > \|\mathbf{g} - \mathbf{1}_m\|_\infty$ ,  $\rho < 1$  that quantifies the deviation of the gains with respect to the ideal case in which they are all equal. Note that  $\rho < 1$  also avoids that any component of  $\mathbf{g}$  is null, which would correspond to losing all measurements from the corresponding sen-



**Fig. 1.** Empirical phase transition of Algorithm 1 as  $n$  increases (top to bottom) and  $k$  increases (left to right), as a function of  $\frac{m}{k}, p$ . We report the estimated contour levels of the probability of successful recovery, as it exceeds the value indicated above of each curve.

sor. Hence, the gains will be inside<sup>2</sup>  $\mathcal{G}_\rho := \mathbf{1}_m + \rho \mathbb{B}_{\ell_\infty}^m \cap \mathbf{1}_m^\perp$ , i.e., on a subset  $\mathcal{G}_\rho \subset \Pi_+^m$ . It is in this closed convex set that we search for  $\mathbf{g}$ . To do so, we start from a point in  $\mathcal{G}_\rho$  and compute the projected gradient with respect to  $\gamma$ ,

$$\nabla_\gamma^\perp f(\xi, \gamma) = \frac{1}{mp} \sum_{l=1}^p \mathbf{P}_{\mathbf{1}_m^\perp} \text{diag}(\mathbf{A}_l \xi) (\text{diag}(\gamma) \mathbf{A}_l \xi - \mathbf{y}_l). \quad (5)$$

This ensures that the steps are taken on  $\mathbf{1}_m^\perp$ . In theory, we would have to use the projection operator  $\mathcal{P}_{\mathcal{G}_\rho}$  to ensure that a gradient step still belongs to this convex set; however, when we start from an initialisation  $\gamma_0 := \mathbf{1}_m$ , we have observed that the algorithm will remain inside  $\mathcal{G}_\rho$  when convergent or, conversely, diverge independently of the presence of  $\mathcal{P}_{\mathcal{G}_\rho}$ . Thus, we will not practically use this projector, while it will be necessary for developing some theory as in [4].

Secondly, as typically done in greedy algorithms, instead

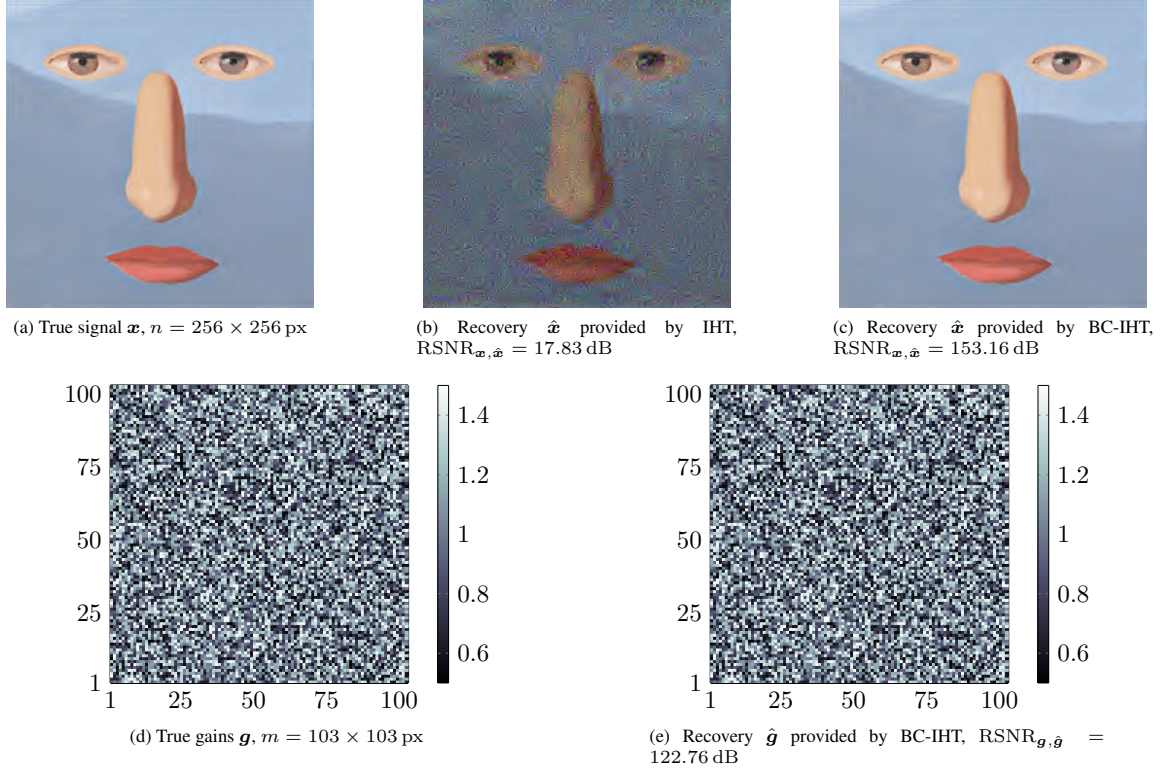
<sup>2</sup> $\mathbf{1}_m^\perp$  is the orthogonal complement of  $\mathbf{1}_m$ , i.e., all vectors whose entries sum to 0. Its projection matrix is  $\mathbf{P}_{\mathbf{1}_m^\perp} = \mathbf{I}_m - \frac{\mathbf{1}_m \mathbf{1}_m^\top}{m}$ .

of adopting a proxy for sparsity such as the  $\ell_1$ -norm we iteratively enforce it by evaluating the gradient

$$\nabla_\xi f(\xi, \gamma) = \frac{1}{mp} \sum_{l=1}^p \mathbf{A}_l^\top \text{diag}(\gamma) (\text{diag}(\gamma) \mathbf{A}_l \xi - \mathbf{y}_l) \quad (6)$$

and applying after each gradient step the *hard thresholding* operator  $\mathcal{H}_k$ , which sets all but the  $k$  largest-magnitude entries of the argument to 0. This operator is at the heart of Iterative Hard Thresholding (IHT, [26]) and allows us to enforce sparsity in the signal domain. Finally, as in [4] we choose an initialisation  $\xi_0 := \frac{1}{mp} \sum_{l=1}^p (\mathbf{A}_l)^\top \mathbf{y}_l$  that is an unbiased estimator of  $\mathbf{x}$ , i.e., as  $p \rightarrow \infty$  we have that  $\xi_0 \rightarrow \mathbb{E} \xi_0 \equiv \mathbf{x}$ .

With all the previous considerations, we approach our version of Blind Calibration with Iterative Hard Thresholding (BC-IHT), as summarised in Algorithm 1. The line searches reported in step 3 can be computed in closed form, and they accelerate in a sub-optimal fashion the algorithm. The step size could be optimised over the non-linear cost: this may yield faster convergence (see, e.g., [27]), but will be the sub-



**Fig. 2.** A numerical example of blind calibration for compressive imaging; the test image is a detail of “Tous les jours”, René Magritte, 1966, © Charly Herscovici, with his kind authorization - c/o SABAM-ADAGP, 2011. The artwork was retrieved at wikiart.org and is intended for fair use. A comparison of the original and retrieved signal and gains is reported in a-c and d-e, respectively.

ject of a future improvement of BC-IHT.

### 3. EMPIRICAL PHASE TRANSITION

We here propose an extensive experimental assessment of the phase transition of BC-IHT. We explore the effect of the problem dimensions in (1) on the successful recovery of both the signal and the gains, by varying  $n = \{2^9, 2^{10}\}$ ,  $k = \{2^5, 2^6, 2^7\}$  and  $m = \lceil \{2, 2^{\frac{5}{4}}, \dots, 2^5\} \cdot k \rceil$ ,  $p = \lceil \{2, 2^{\frac{5}{4}}, \dots, 2^5\} \rceil$  and generating 144 random instances of the problem for each of the configurations. In detail,  $\mathbf{x} \sim_{\text{i.i.d.}} \mathcal{N}^n(0, 1)$  is drawn as a standard Gaussian random vector<sup>3</sup>;  $\mathbf{g}$  is drawn uniformly at random on  $\mathcal{G}_\rho$  for  $\rho = \frac{1}{2}$ ;  $\mathbf{A}_l \sim_{\text{i.i.d.}} \mathcal{N}^{m \times n}(0, 1)$  are drawn as i.i.d. Gaussian random matrices. We then let the algorithm run given  $\mathbf{y}_l$  and  $\mathbf{A}_l$ ,  $l \in [p]$  up to a relative change of  $10^{-7}$  in the signal and gain updates. Then, we measure the probability of successful

recovery

$$P_\zeta(n, k, m, p) := \mathbb{P} \left[ \max \left\{ \frac{\|\hat{\mathbf{g}} - \mathbf{g}\|_2}{\|\mathbf{g}\|_2}, \frac{\|\hat{\mathbf{x}} - \mathbf{x}\|_2}{\|\mathbf{x}\|_2} \right\} < \zeta \right]$$

on the trials, with  $\zeta = -60$  dB (this corresponds to an early termination of the algorithm: when convergent it will reach the exact solution, provided we let it run for a sufficient number of iterations). The results are reported in Figure 1 in terms of the estimated contour levels of  $P_\zeta$ , in function of  $\log_2 \frac{m}{k}$  and  $\log_2 p$ .

While a theoretical sample complexity result that grants provable convergence is under study, we can already appreciate that the effect of increasing  $n$  for fixed sparsity levels has a mild effect on the region in which  $P_\zeta > 0.95$ , while it does sharpen the transition region as typically observed in standard CS. Moreover, we can appreciate the impact of increasing  $k$  on the transition region while keeping the ratios  $\frac{m}{k}$  fixed: for larger values of  $k$ , the region in which the algorithm fails to converge almost surely is rapidly reduced. Moreover, we reported in red the curve corresponding to  $mp = C(k + m)$  (i.e.,  $\log_2 p = \log_2 C(1 + \frac{k}{m})$ ) for some  $C > 0$ , which roughly follows the contours of the experimental results.

We highlight that all the empirical evidence collected in

<sup>3</sup>The convention  $\mathcal{N}^{m \times n}(\mu, \sigma^2)$  indicates the generation of an  $m \times n$  matrix (or vector) with i.i.d. Gaussian entries having mean  $\mu$  and variance  $\sigma^2$ .

this context correctly suggests that  $p > 1$ : this agrees with our previous finding that  $p \gtrsim \log m$  (see [4, Proposition 2]), *i.e.*, if no structure is leveraged on the gains  $\mathbf{g}$  more than one snapshot will always be needed for the algorithm to collect a sufficient amount of information on  $\mathbf{g}$ .

Thus, by interpreting the results, we can expect that if  $m = 4k$  (a widely used rule of thumb in CS), our blind calibration method will converge for most instances of (1) and  $\rho < 1$ , once we let  $p > 4$  we will be able to recover both  $(\mathbf{x}, \mathbf{g})$ . If furthermore  $k$  is sufficiently low, the total under-sampling factor  $\frac{mp}{n}$  will be below 1.

#### 4. BLIND CALIBRATION FOR COMPRESSIVE IMAGING

We now proceed to apply BC-IHT in a practical case, in which we process a high-dimensional red-green-blue (RGB) image  $\mathbf{x}$  of dimension  $n = 256 \times 256$  px, which is made sparse on a Daubechies-4 orthonormal wavelet basis with only  $k = 1800$  non-zero coefficients. Then  $\mathbf{x}$  is acquired by means of Gaussian random sensing matrices  $\mathbf{A}_l$ ,  $l \in [p]$ . This experiment could be carried out with other sub-Gaussian matrix ensembles such as Bernoulli sensing matrices, with the results being substantially unaltered. Since the sparsity level of the chosen test image is high, we can simulate its acquisition with a sensor array of  $m = 103 \times 103$  px ( $m \approx 6k$ ) and use  $p = 5$  snapshots to meet the requirements of our method; thus  $\frac{mp}{n} \approx 0.8$ , and once the gains are retrieved this CS scheme could revert to  $\frac{m}{n} \approx 0.16$  while benefiting from the improved model accuracy provided by blind calibration. As for the gains, we set  $\rho = \frac{1}{2}$  and draw  $\mathbf{g}$  uniformly at random from  $\mathcal{G}_\rho$ .

We then run BC-IHT on each of the RGB channels separately, until the relative change in the signal and gain estimates falls below  $10^{-7}$ ; the quality and data reported below are the worst case among the colour channels. This causes the algorithm to run for 884 iterations, achieving a high-quality estimate having  $\text{RSNR}_{\mathbf{x}, \hat{\mathbf{x}}} = -20 \log_{10} \frac{\|\hat{\mathbf{x}} - \mathbf{x}\|_2}{\|\mathbf{x}\|_2} = 153.16$  dB and  $\text{RSNR}_{\mathbf{g}, \hat{\mathbf{g}}} = -20 \log_{10} \frac{\|\hat{\mathbf{g}} - \mathbf{g}\|_2}{\|\mathbf{g}\|_2} = 122.76$  dB. The quality of the estimates can be observed in Figure 2c and 2e.

To see the beneficial effect of blind calibration, we use the accelerated version of IHT [27] given the exact sparsity level  $k$ , the snapshots  $\mathbf{y}_l$  and the corresponding sensing matrices, which form a standard CS model when concatenated vertically. Hence, accelerated IHT attempts to recover an estimate  $\hat{\mathbf{x}}$  while neglecting the model error. The algorithm converges in only 29 iterations to a local minimiser  $\hat{\mathbf{x}}$ , whose  $\text{RSNR}_{\mathbf{x}, \hat{\mathbf{x}}} = 17.83$  dB. The modest performances can be seen directly in Figure 2b.

No comparison with other blind calibration algorithms has reported since the peculiar choice of using a single sparse input and multiple snapshots is specific to our framework. Nevertheless, we note that (i) the computational complexity

of our algorithm is competitively low, as it amounts to that of IHT plus an additional projected gradient step in the gain domain per iteration; (ii) just as a proof of convergence for IHT to a local minimiser has been devised, we expect to have provable convergence results in the same fashion, which will lead to a bound on the sample complexity that ensures the retrieval of the exact solution.

#### 5. CONCLUSION

We proposed a novel approach to blind calibration based on the use of snapshots with multiple draws of the random sensing operator, and on a greedy algorithm which enforces sparsity on the steps resulting from gradient descent on a non-convex objective. Our approach is capable of achieving, within a few snapshots, perfect recovery of the signal and gains in a computationally efficient fashion. Hence, we conclude that when sensor calibration is a cause of concern in a sensing scheme, introducing a modality that follows (1) and using our method could be a viable option to cope with model errors.

We envision that our method may be used both for blind calibration of imaging sensors, as well as distributed sensor arrays or networks if suitably modified to allow for compressive sensing. While we presented empirical evidence on the phase transition of our algorithm, a more rigorous convergence guarantee is under study and will be the subject of a future communication.

#### 6. REFERENCES

- [1] D. L. Donoho, “Compressed sensing,” *IEEE Transactions on information theory*, vol. 52, no. 4, pp. 1289–1306, 2006.
- [2] M. A. Herman and T. Strohmer, “General deviants: An analysis of perturbations in compressed sensing,” *IEEE Journal of Selected Topics in Signal Processing*, vol. 4, no. 2, pp. 342–349, 2010.
- [3] V. Cambareri, M. Mangia, F. Pareschi, R. Rovatti, and G. Setti, “Low-complexity multiclass encryption by compressed sensing,” *IEEE Transactions on Signal Processing*, vol. 63, no. 9, pp. 2183–2195, 2015.
- [4] V. Cambareri and L. Jacques, “A Non-Convex Blind Calibration Method for Randomised Sensing Strategies,” in *4<sup>th</sup> International Workshop on Compressed Sensing Theory and its Applications to Radar, Sonar, and Remote Sensing (CoSeRa 2016)*, Aachen, Germany, Sep. 2016.
- [5] —, “Through the Haze: A Non-Convex Approach to Blind Calibration for Linear Random Sensing Models,” 2016, submitted to Information and Inference: A Journal of the IMA.

- [6] R. Vershynin, "Introduction to the non-asymptotic analysis of random matrices," in *Compressed Sensing: Theory and Applications*. Cambridge University Press, 2012, pp. 210–268.
- [7] J. Romberg, "Compressive sensing by random convolution," *SIAM Journal on Imaging Sciences*, vol. 2, no. 4, pp. 1098–1128, 2009.
- [8] S. Bahmani and J. Romberg, "Compressive deconvolution in random mask imaging," *IEEE Transactions on Computational Imaging*, vol. 1, no. 4, pp. 236–246, 2015.
- [9] J. P. Dumas, M. A. Lodhi, W. U. Bajwa, and M. C. Pierce, "Computational imaging with a highly parallel image-plane-coded architecture: challenges and solutions," *Opt. Express*, vol. 24, no. 6, pp. 6145–6155, Mar. 2016. [Online]. Available: <http://www.opticsexpress.org/abstract.cfm?URI=oe-24-6-6145>
- [10] A. Liutkus, D. Martina, S. Popoff, G. Chardon, O. Katz, G. Lerosey, S. Gigan, L. Daudet, and I. Carron, "Imaging with nature: Compressive imaging using a multiply scattering medium," *Scientific reports*, vol. 4, 2014.
- [11] M. M. Hayat, S. N. Torres, E. Armstrong, S. C. Cain, and B. Yasuda, "Statistical algorithm for nonuniformity correction in focal-plane arrays," *Applied Optics*, vol. 38, no. 5, pp. 772–780, 1999.
- [12] L. Balzano and R. Nowak, "Blind calibration of networks of sensors: Theory and algorithms," in *Networked Sensing Information and Control*. Springer, 2008, pp. 9–37.
- [13] J. Lipor and L. Balzano, "Robust blind calibration via total least squares," in *Acoustics, Speech and Signal Processing (ICASSP), 2014 IEEE International Conference on*. IEEE, 2014, pp. 4244–4248.
- [14] H. Zhu, G. Leus, and G. B. Giannakis, "Sparsity-cognizant total least-squares for perturbed compressive sampling," *IEEE Transactions on Signal Processing*, vol. 59, no. 5, pp. 2002–2016, 2011.
- [15] J. Parker, V. Cevher, and P. Schniter, "Compressive sensing under matrix uncertainties: An Approximate Message Passing approach," in *2011 Conference Record of the Forty Fifth Asilomar Conference on Signals, Systems and Computers (ASILOMAR)*, Nov. 2011, pp. 804–808.
- [16] F. Krzakala, M. Mezard, and L. Zdeborova, "Phase diagram and approximate message passing for blind calibration and dictionary learning," in *2013 IEEE International Symposium on Information Theory Proceedings (ISIT)*, Jul. 2013, pp. 659–663.
- [17] C. Bilen, G. Puy, R. Gribonval, and L. Daudet, "Convex Optimization Approaches for Blind Sensor Calibration Using Sparsity," *IEEE Transactions on Signal Processing*, vol. 62, no. 18, pp. 4847–4856, Sep. 2014.
- [18] B. Friedlander and T. Strohmer, "Bilinear compressed sensing for array self-calibration," in *2014 48th Asilomar Conference on Signals, Systems and Computers*, Nov. 2014, pp. 363–367.
- [19] C. Schülke, F. Caltagirone, and L. Zdeborová, "Blind sensor calibration using approximate message passing," *Journal of Statistical Mechanics: Theory and Experiment*, vol. 2015, no. 11, p. P11013, 2015.
- [20] S. Ling and T. Strohmer, "Self-calibration and biconvex compressive sensing," *Inverse Problems*, vol. 31, no. 11, p. 115002, 2015.
- [21] A. Ahmed, B. Recht, and J. Romberg, "Blind deconvolution using convex programming," *IEEE Transactions on Information Theory*, vol. 60, no. 3, pp. 1711–1732, 2014.
- [22] E. Candès, X. Li, and M. Soltanolkotabi, "Phase Retrieval via Wirtinger Flow: Theory and Algorithms," *IEEE Transactions on Information Theory*, vol. 61, no. 4, pp. 1985–2007, Apr. 2015.
- [23] C. D. White, S. Sanghavi, and R. Ward, "The local convexity of solving systems of quadratic equations," *arXiv:1506.07868 [math, stat]*, Jun. 2015, arXiv: 1506.07868.
- [24] X. Li, S. Ling, T. Strohmer, and K. Wei, "Rapid, Robust, and Reliable Blind Deconvolution via Nonconvex Optimization," *arXiv preprint arXiv:1606.04933*, 2016.
- [25] A. Ahmed, F. Krahmer, and J. Romberg, "Empirical Chaos Processes and Blind Deconvolution," *arXiv preprint arXiv:1608.08370*, 2016.
- [26] T. Blumensath and M. E. Davies, "Iterative hard thresholding for compressed sensing," *Applied and Computational Harmonic Analysis*, vol. 27, no. 3, pp. 265–274, 2009.
- [27] T. Blumensath, "Accelerated iterative hard thresholding," *Signal Processing*, vol. 92, no. 3, pp. 752–756, 2012.